Transport through a 1D Mott-Hubbard insulator of finite length.

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Transport through a 1D Mott-Hubbard insulator of a finite length L is studied beyond perturbative approach. At special value of the low energy constant of the interaction we have mapped the problem onto the exactly solvable models and found current vs. voltage V at high temperature $T > max(m, T_L)$ and at low energy $T, V < T_L$ ($T_L = v_c/L$; v_c : charge velocity). The result shows that for the strong interaction creating a large Mott-Hubbard gap $2m \gg T_L$ inside the wire, the transport is suppressed near half-filling everywhere inside the gap except for an exponentially small region of $V, T < T_L exp(-2m/T_L)$.

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Motivated by current experiments [1] of Tarucha's group, a few works recently appeared where an effect of Umklapp processes on quantum transport through a 1D wire has been studied. Previously, it has been already shown by experiment [2] and in theory [3–5] that there occurs no renormalization of the conductance due to the electron-electron interaction. Nonetheless, this renormalization has been observed [6]. A recent calculation [8] based on Kubo formalism [7] studied the effect of the Umklapp backscattering and concluded that it affects the zero temperature conductance. This contradicts the perturbative results of Ref. [9,10], where the quantized conductance $G = e^2/(\pi\hbar)$ has been obtained at $T = 0$ (below we use the units, where $e = \hbar = 1$). Another point of view suggests [11,12] that in a non-perturbative case of large Mott-Hubbard gap $2m \gg v_c/L \equiv T_L$ (L: length of the wire; v_c: charge velocity) the conductance is strongly suppressed at low energy inside the gap similar to the band gap case.

In this paper, to clear up the difference between the Mott-Hubbard insulator and the band gap one, we map the problem at low energies and at high temperature onto the exactly solvable models making use of a free fermion value of the constant g of the forward scattering inside the wire. The results are shown in Figs. 1 and 2. At low energies when $T, V \ll T_L$ (T: temperature; V: voltage), we have found that a new energy scale $T_x \propto T_L exp[-2\sqrt{m^2-\mu^2}/T_L]$ appears in the system if the chemical potential μ is small enough: $\sqrt{m^2-\mu^2}/T_L \gg 1$. Below T_x the conductance is not suppressed contrary to the expectation of Starykh and Maslov [12] and the current increases linearly. Above this energy the current saturates and the conductance goes down as T_x/T reaching small values $\approx exp[-2\sqrt{m^2-\mu^2}/T_L]$ at $T \approx T_L$. At high temperature $T \gg T_L, m$ we confirmed the asymptotical behavior of conductance: $G = (1 - cst\frac{m}{T}(1 + \cosh\frac{\mu}{T})^{-1})/\pi$ predicted before for Mott-Hubbard insulator in perturbative regime of small gap [9,10] where the constant depends on m/T_L.

A brief physical explanation to these results follows. At low energies $T < T_L$ and $\mu \ll m$ the charge field is quantized inside the wire at its values related to the degenerate sin-Gordon vacua. Rare low energy excitations tunnel through the wire with the amplitude $\propto exp[-m/T_L]$ as (anti)solitons switching the quantized value of the field. The whole process of tunneling, however, includes transformation of the reservoir electron into the sin-Gordon quasiparticles and back. This transformation results in a non-trivial scaling dimension of the tunneling operator equal to $1/2$ for the Mott-Hubbard insulator connected to the Fermi liquid reservoirs independently of any parameters. In the case of the band insulator, this dimension is marginal ($= 1$): the transformation is trivial and does not introduce additional energy dependence. The infrared relevantness of the tunneling with the $1/2$ dimension brings out above resonance at zero energy. Meanwhile, the exponentially small tunneling amplitude specifies the narrow width of this resonance equal to the crossover energy. Increase of $|\mu|$ favours tunneling of the quasiparticles of the same sort and ultimately produces their finite density in the wire. Then the interaction between these quasiparticles described with the two-particle S-matrices [13] dependent on g emerges. At low momenta the S-matrix for the quasiparticles of the same sort is inevitably free fermion like, as at $g = 1/2$. It manifests in the renormalization group (RG) flow derived from the Bethe anzats solution for the massive phase [14] of the sin-Gordon model and in the exponent calculated for the Tomonaga Luttinger liquid (TLL) phase at low density [15]. Increase of T, on the other hand, is expected to entail, first, a thermally activated behavior of the conductance $\propto exp[-2m/T]$ at $T_L < T < m$ [16] and then a power law dependence at $m < T$. Since the effective value of g, in general, scales with energy, the $1/T$ dependence we found for $g = 1/2$ may vary at higher energies $T \gg m$ depending on the high energy value of g.

Transport through the finite length wire under a constant voltage V between the left and right leads could be described in the inhomogeneous Tomonaga-Luttinger liquid model (TLL) with the Lagrangian [9,17]: $\int dx\{\sum_b \mathcal{L}_b(x, \phi_b, \partial_t\phi_b) + \mathcal{L}_{bs}(x, Vt, \phi_c, \phi_s)\}$. The bosonic fields $\phi_b(x,t), b = c, s$ relate to the deviations of

the charge and spin densities from their average values as following: $\rho_b(x,t) = (\partial_x \phi_b(x,t))/(\sqrt{2}\pi)$, respectively. The first part of the Lagrangian describes a free electron motion modified by the forward scattering interaction. The second part of the Lagrangian introduces backscattering inside the wire. Only its term corresponding to the Umklapp process of four Fermi momenta transfer is important near half-filling. This term does not involve the spin field. Therefore, our consideration will be restricted to the charge field only. For the clean wire this field is characterized by the Lagrangian:

$$\int dx \mathcal{L}_t = \int dx \big[\frac{v_c(x)}{2g(x)}\{\frac{1}{v_c^2}\left(\frac{\partial_t \phi_c(t,x)}{\sqrt{4\pi}}\right)^2 - \left(\frac{\partial_x \phi_c(t,x)}{\sqrt{4\pi}}\right)^2\} - \frac{E_F^2 U}{v_F}\varphi(x)\cos(4k_F x + 2Vt + \sqrt{2}\phi_c(t,x))\big] \qquad (1)$$

where $\varphi(x) = \theta(x)\theta(L-x)$ specifies a one channel wire of the length L adiabatically attached to the leads $x > L, x < 0$ and $v_F(E_F)$ denotes the Fermi velocity(energy) in the channel. The parameter $4k_F$ varies the chemical potential $\mu = 2k_F v_c = E_{thr}$ of the wire from its zero value at half-filling. Outside the Hubbard gap this parameter coincides with the momentum transfered by the backscattering: four Fermi momenta minus a vector of the reciprocal lattice, and relates the present results to the perturbative ones [9]. The constant of the forward scattering varies from $g_c(x) = g$ inside the wire ($x \in [0,L]$) to $g_c(x) \equiv g_\infty = 1$ inside the leads, and the Umklapp scattering of the strength U is introduced inside the wire. The charge velocity $v_c(x)$ changes from v_F outside the wire to a some constant v_c inside it. In the absence of the Umklapp scattering, $v_c \simeq v_F/g$ and $0 < g < 1$ is determined by the forward scattering amplitude of the bare short range interaction between electrons. Approaching the half-filling put the Umklapp scattering on. It entails an essential renormalization of the low energy value of g, which flows to its free fermion value $g = 1/2$ in the massive phase [14] ($\mu < m$) where the coefficient of the cos-term scales to $\simeq m^2$ and on approaching this phase [15] $|\mu| \searrow m$. This value of g will be assumed below. The zero frequency current through the wire equals $I = V/\pi + <\hat{I}_{bsc}>$, where the backscattering current [17] is $\hat{I}_{bsc} = -2E_F^2 U/v_F \int_0^L dx \sin(4k_F x + 2Vt + \sqrt{2}\phi_c(x))$. It will be shown later that $2\pi E_F U$ is a half gap m, opened by the backscattering (1) in the charge mode spectrum inside the wire.

1.*High temperatures* $T > T_L, m$ - The average backscattering current $<\hat{I}_{bsc}> = \int D\phi \hat{I}_{bsc} exp\{i\int dt \int dx(\mathcal{L}_c + \mathcal{L}_{bsc})\}$ can be written as a formal infinite series in U. Each term of it is an integral of product of the free bosonic correlators $< exp\{i\sqrt{2}\phi(x,t)\}exp\{-i\sqrt{2}\phi(y,0)\} >$. Such a correlator approaches its uniform TLL expression when $x, L-x, y, L-y \gg v_c/T$. Substitution of this form into above series allowed us [9] to find L-proportional part of the backscattering current neglecting the boundary contribution in the perturbative case. However, the problem is not perturbative, in general, due to a finite gap $2m$ creation. Therefore, application of the uniform correlator will give us a part of the backscattering current $\propto min(L, v_c/m)$ with the relative error $O(max(T_L, m)/T)$, which is of the order of ratio of the border piece $\propto v_c/T$ to the essential part of the "bulk" one. This relates to the high-temperature asymptotics of the whole current.

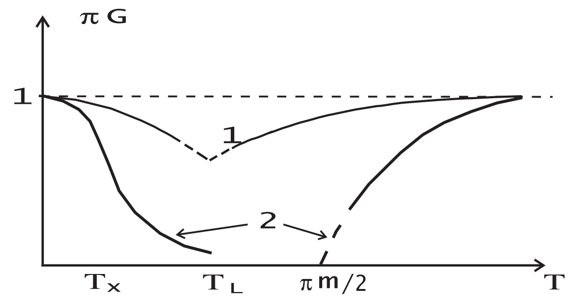

FIG. 1. Schematic linear bias conductance G vs. temperature near half-filling $\mu \ll T_L$: curve 1 for the weak interaction $m \ll T_L$, curve 2 for the strong one.

Calculation of above series with the uniform TLL correlator is equivalent to expanding the value $g = 1/2$ into the leads. Following Luther and Emery [18] we map this bosonic Lagrangian onto the free massive fermion one [11,12] with the density of Lagrangian

$$\mathcal{L}_F = i \sum_{a=R,L=\pm} \psi_a^+ (\partial_t \pm v_c \partial_x)\psi_a - m\varphi(x)[e^{i(4k_F x + 2Vt)}\psi_L^+(x,t)\psi_R(x,t) + h.c.] \qquad (2)$$

Here $\psi_{R(L)}$ is right (left) chiral fermion field. The fermionized backscattering current $\hat{I}_{bsc} = 2im\int_0^L dx[exp\{i(4k_F x + 2Vt)\}\psi_L^+(x,t)\psi_R(x,t) - h.c.]$ is the doubled backscattering current for the fermions [17] under doubled voltage. To find its average we just need to know the fermionic reflection coefficient R as a function of dimensionless energy $\omega = \varepsilon/m$:

$$R(\omega) = \frac{\sin^2(\sqrt{\omega^2-1}\bar{t}_L)}{(\omega^2-1)+\sin^2(\sqrt{\omega^2-1}\bar{t}_L)} \tag{3}$$

where $\bar{t}_L \equiv m/T_L$ denotes the dimensionless traversal time. The analytical continuation is assumed for $|\omega| < 1$. Since the chemical potential for the right/left chiral fermions is $\mu \pm V$, respectively, the total current can be expressed as

$$I = \frac{V}{\pi} - \frac{m\sinh(V/T)}{\pi}\times \int d\omega \frac{R(\omega)}{\cosh((m\omega-\mu)/T)+\cosh(V/T)} \tag{4}$$

Only the leading term in $max(m, T_L)/T$ of the right hand side of (4) is meaningful. Extracting it, we find the high-temperature asymptotics as following

$$I = \frac{V}{\pi} - \frac{m}{\pi}\frac{\sinh(V/T)B(mt_L)}{\cosh((\mu)/T)+\cosh(V/T)} \tag{5}$$
$$B(x) \equiv \int d\omega \frac{\sin^2(\sqrt{\omega^2-1}x)}{(\omega^2-1)+\sin^2(\sqrt{\omega^2-1}x)}$$

where function $B(x)$ increases as πx at small $x > 0$ and approaches the constant $\simeq \pi$ at $x \gg 1$. Accuracy of this calculation of (4) may be written as a factor $1 + O(max(m, T_L)/max(T, V))$ to (5) if $|\mu| \ll max(T, V)$ or as $1 + O([max(m, T_L)/max(T, V)](max(T, V)/\mu)^2 e^{|\mu|/T})$, otherwise. The high-temperature conductance (Fig.1)

$$G = \frac{1}{\pi}\left(1 - \frac{m}{T}\frac{B(m/T_L)}{1+\cosh(\mu/T)}\right) \tag{6}$$

approaches zero at $T \approx m$ if the gap is large enough $m/T_L \gg 1$ and $|\mu| < m$.

2.*Low energies* $T, V << T_L$ - To find a low energy model for our problem we have to integrate out all high energy modes. We will try to escape direct integration following Wiegmann's effective way of constructing the Bethe-ansatz solvable models [19] for the Kondo problem and for the screening of a resonant level [19,20]. First, let us substitute $\phi/\sqrt{2}$ instead of ϕ in (1). It makes fermions interacting inside the leads and non-interacting inside the wire. Their passage through the wire may be described with the one-electron S-matrix dependent on the electron momentum. The interaction between electrons in the leads with some two-particle S-matrix. Then the solution could be constructed if the proper commutation relations between the S-matrices are met. Being interested in the variation of energy less then T_L around the Fermi level, one can simplify the solution keeping the one-particle S-matrix constant equal to its value on the Fermi level. It leads us to the problem of one impurity in the TLL.

For the weak backscattering, the Lagrangian of this problem can be written as

$$\int dx\mathcal{L}_t = \int dx \frac{v_F}{2}\{\frac{1}{v_F^2}\left(\frac{\partial_t\phi_c(t,x)}{\sqrt{4\pi}}\right)^2 - \left(\frac{\partial_x\phi_c(t,x)}{\sqrt{4\pi}}\right)^2\} - \frac{YT_L u}{\pi v_F}\cos(2Vt + \sqrt{2}\phi_c(t,0)) \tag{7}$$

where we rescaled ϕ back and introduced a new energy cut-off parameter YT_L with dimensionless constant Y which will be specified later. Parameter u is related to the weak reflection coefficient as:$u^2 = v_F^2 R(\mu/m)$. For the strong backscattering the tunneling Hamiltonian approach may be applied [21]. It was associated [25] to the dual representation using the field θ mutually conjugated to ϕ : $[\theta_\sigma(x), \phi_\sigma(y)] = i2\pi sgn(x-y)$. The appropriate Lagrangian reads

$$\int dx\mathcal{L}_t = \int dx \frac{v_F}{2}\{\frac{1}{v_F^2}\left(\frac{\partial_t\theta_c(t,x)}{\sqrt{4\pi}}\right)^2 - \left(\frac{\partial_x\theta_c(t,x)}{\sqrt{4\pi}}\right)^2\} - \frac{YT_L u'}{\pi v_F}\cos(Vt + \theta_c(t,0)/\sqrt{2}) \tag{8}$$

with $u'^2 = v_F^2(1 - R(\mu/m))$ proportional to the free massive fermion transmittance and the voltage multiplied by g factor [24]. Both these Lagrangian are, indeed, equivalent [22] if interaction dependent relation between u and u' is met [23,24]. The above model (7) or(8) characterizes the point scatterer of any backscattering strength at low energy [25]. Although, the exact relation between u or u' and the bare parameters of the scatterer remains unknown. Our problem is dually symmetrical to that of Kane and Fisher: suppression of the direct current in their problem equals suppression of the backscattering one in our case. This correspondence allows us to re-write their solution [23,25] as follows:

$$I = \frac{T_x}{\pi} Im\psi(\frac{1}{2} + \frac{T_x + iV}{\pi T})$$
$$I = \frac{T_x}{\pi}\arctan(V/T_x), \quad T = 0 \tag{9}$$
$$G = \frac{T_x}{\pi^2 T}\psi'(\frac{1}{2} + \frac{T_x}{\pi T})$$

where ψ denotes the digamma function and satisfies: $\psi'(1/2) = \pi^2/2$, $\psi'(x) \propto 1/x$, $x \to \infty$, and a new energy scale T_x [24] varies from $T_x = YT_L\sqrt{4R}$ at the weak backscattering (7) to $T_x = YT_L(1-R)/\pi$ at the strong

one (8).

Let us, first, compare this result with the perturbative one [9,10]. The latter was derived making use of the long-time asymptotics for the correlator:

$$< e^{i\phi_c(x,t)} e^{-i\phi_c(y,0)} >= cst\big(\alpha T_L\big)^{2g}\big(\frac{(\pi T/T_L)^2}{sinh(\pi(t-i\alpha)T)sinh(\pi(-t+i\alpha)T)}\big)F(x)F(y) \qquad (10)$$

where $\alpha = 1/E_F$ and F was simplified: $F(x) = (x/L)^{gr}\prod_{\pm,m=1}^{\infty}(m\pm x/L)^{gr^{2m\pm1}} \approx cst'\times[x(L-x)/L^2]^{gr}$. $r = (1-g)/(1+g)$ is the reflection coefficient of the charge wave scattering on the contacts of the wire with the leads. One can see that the current calculated with this form of the correlator will be consistent with the Lagrangian (7) if we choose Y matching

$$\frac{Y}{\sin(\mu/T_L)} = \frac{cst}{\sqrt{\pi}\Gamma(1+r)}\frac{(2\mu/T_L)^{r-1/2}}{J_{r+1/2}(\mu/T_L)} \qquad (11)$$

This specifies Y as a constant of the order of 1 for $|\mu|/T_L < 1$. Then the solution (9) coincides with the perturbative result [9,10], that is $I - V/\pi \propto -V^3$ and $G - 1/\pi \propto -T^2$.

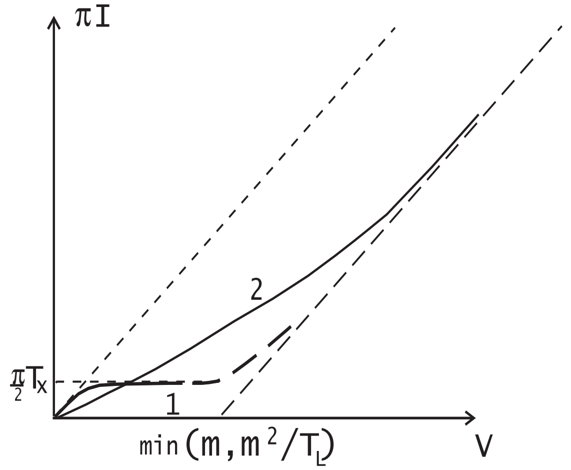

FIG. 2. Schematic current I vs. voltage V near half-filling $\mu \ll T_L$: curve 1 is zero temperature dependence, curve 2 is the high temperature $T \gg T_L$ one, the dashed lines are the low voltage $I = V/\pi$ and high voltage asymptotics.

Turning to the case $m/T_L \geq 1$ we cannot use the perturbative expression (11) anymore: the perturbative series is not convergent due to a finite gap creation. Then above non-perturbative consideration is necessary. Application of the solution (9) in this case reveals a quite remarkable property of low energy transport through the Mott-Hubbard insulator. There is an exponentially small value of $T_x = (1 - R(\mu/m))YT_L/\pi \propto T_L exp(-2m/T_L)$ for $\mu \ll m$. Hence, the zero temperature current I (Fig.2) is not suppressed for the voltage less than T_x and saturates at $T_x/2$ value when $T_x < V < T_L$. Similarly, the conductance (Fig.1) displays a small decrease $\propto T^2$ below its zero temperature value $1/\pi$ with increase of T for $T < T_x$ and approaches its exponentially small asymptotics $G = T_x/(4T) \propto exp(-2m/T_L)T_L/T$ above $T_x < T < T_L$. As $|\mu|$ increases, the reflection coefficient $R(\mu/m)$ on the Fermi level goes down and T_x exceeds T_L, finally, approaching its weak backscattering value $2T_x = YT_L/\sqrt{R(\mu/m)}$, where the perturbative consideration is applicable.

In summary, we studied transport through a 1D Mott-Hubbard insulator beyond perturbative approach. Assuming that $g = 1/2$ near the half-filling in agreement with the Bethe anztas solutions we mapped the problem onto the exactly solvable models and found current vs. voltage at high temperature $T > max(m, T_L)$ and at low energy $T, V < T_L$. The solution of these models shows, in particularly, that the high-temperature transport through the Mott-Hubbard insulator is similar to the one through the band gap insulator at $g = 1/2$. At low energies, however, there is always a regime where the transport remains unsuppressed in the absence of the impurity backscattering. For the strong interaction resulting in the opening of the large Mott-Hubbard gap, the transport through the wire is suppressed near the half-filling almost everywhere inside the gap except for an exponentially small low energy region $V, T < T_L exp(-m/T_L)$.

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